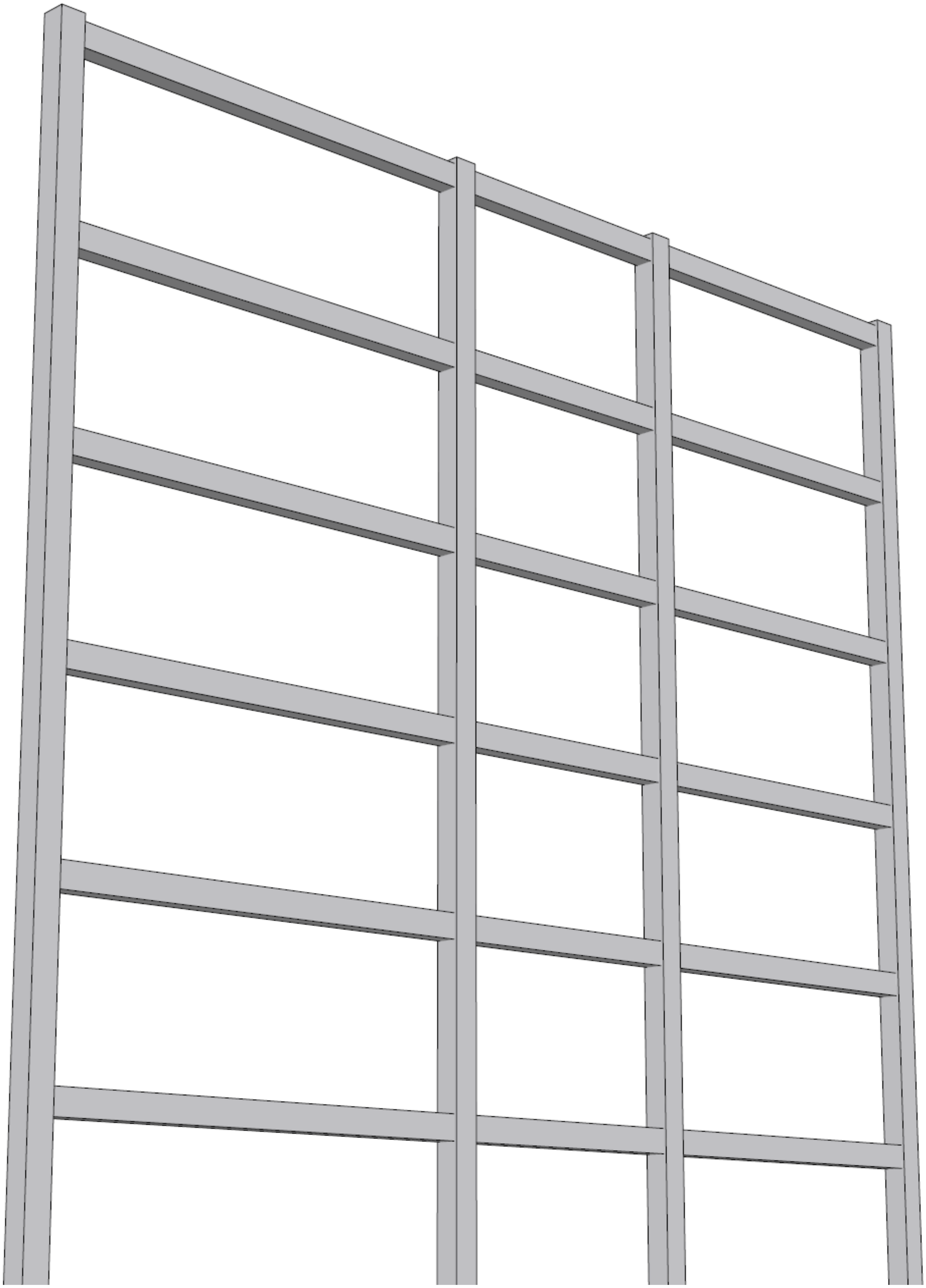
Continuous Beam Design with Moment Redistribution (CSA A23.3-14)



Continuous Beam Design with Moment Redistribution (CSA A23.3-14)

A structural reinforced concrete continuous beam at an intermediate floor level in an exterior frame (spandrel Beam) provides gravity load resistance for the applied dead and live loads.

The required reinforcement areas are determined for this continuous beam after analysis are adjusted and optimized using moment redistribution provisions from CSA A23.3 standard. The results of hand calculations are then compared with numerical analysis results obtained from the spBeam engineering software program.

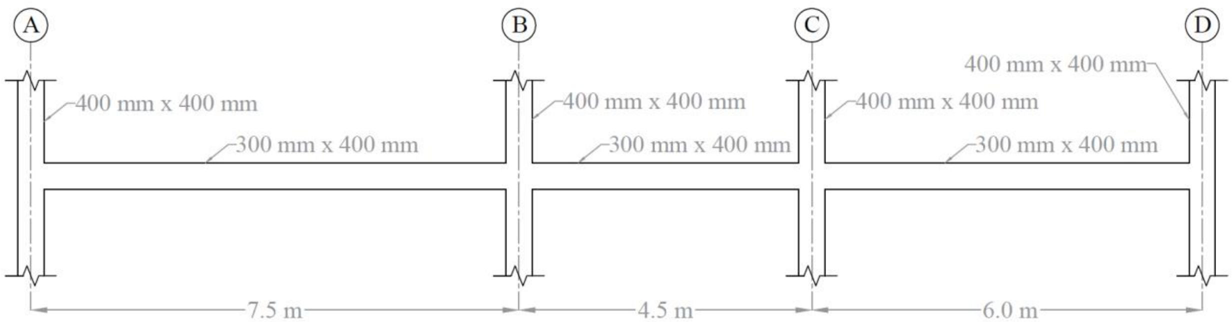


Figure 1 – Reinforced Concrete Continuous Beam at intermediate floor level

Contents

Code

Design of Concrete Structures (CSA A23.3-14) and Explanatory Notes on CSA Group standard A23.3-14 "Design of Concrete Structures"

Reference

PCA Notes on ACI 318-11 Building Code Requirements for Structural Concrete, Twelfth Edition, 2013 Portland Cement Association, Example 8.2

spBeam Engineering Software Program Manual v5.00, STRUCTUREPOINT, 2015

Design Data

f_c' = 25 MPa normal weight concrete (w_c = 24 kN/m^3)

f_y = 400 MPa

Story height =3 m

Columns = 400 mm × 400 mm

Spandrel beam = 400 mm × 400 mm

Dead Loads (DL) = 17 kN/m

Live Loads (LL) = 7 kN/m

Solution

Continuous beams are frequently analyzed and designed using simplified methods such as the approximate frame analysis coefficients provided in CSA A23.3 to approximate the bending moments and shear forces. There are many important limitations to allow the use of coefficients. The factored moment and shear can be determined using the simplified method if the requirements are satisfied: ***CSA A23.3-14 (9.3.1)***

✓ There are at least two spans.

✓ Loads are uniformly distributed.

✓ Members are prismatic.

✓ Factored live load ≤ Twice the factored dead load

X The longer of two adjacent spans does not exceed the shorter by more than 20 percent.

In this example the ratio of the two adjacent spans lengths exceeds 1.2 and coefficients can't be used. Therefore, the analysis of the continuous beam framing into columns must use traditional analysis methods and will be performed using the following steps:

1. Determine the factored loads.
2. Perform the structural analysis using the moment distribution method.
3. Repeat the analysis for each live load pattern to arrive at the enveloped maximum design moments.
4. Apply moment redistribution provisions to get adjusted (reduced) design moments.
5. Determine the required area of steel optimized to the adjusted design moments.

It is important in this example to distinguish between the two terms: **moment distribution** and **moment redistribution**. **Moment distribution** is a structural analysis method for statically indeterminate beams and frames, while **moment redistribution** refers to the behavior of statically indeterminate structures that are not completely elastic, but have some reserve plastic capacity. When one location first yields, further application of load to the structure causes the bending moment to redistribute differently from what a purely elastic analysis would suggest.

Moment redistribution is not commonly used primarily due to the extensive additional and tedious calculations required involving many live load patterns and the iterative nature of the procedure that lends itself to automation as is provided by spBeam engineering software program from StructurePoint.

When permitted, **moment redistribution** is used to reduce total reinforcement required and this example will illustrate the extent of redistribution of bending moments and the corresponding reduction of steel area achievable. Typically, negative moments over supports governs the design of reinforcement and any reduction in the required area of steel at the supports is favorable due to savings in materials, labor, and construction time and effort.

1. Continuous Beam Analysis – Moment Distribution Method

Determine moment distribution factors and fixed-end moments for the frame members. The moment distribution procedure will be used to analyze the frame. Stiffness factors, carry over factors, and fixed-end moment factors for the beams and columns are determined as follows:

Determine the elastic bending moment diagrams for each of the load patterns per CSA and the maximum moment envelope values for all patterns as shown in Table 1. ***CSA A23.3-14 (9.2.3)***

1.1. Load combination

$U = 1.25D + 1.5L$ ***CSA A23.3-14 (Annex C, Table C.1a)***

$w_d = 1.25 \times 17 = 21.25$ kN/m

$w_l = 1.5 \times 7 = 10.5$ kN/m

$w_f = 21.25 + 10.5 = 31.75$ kN/m

1.2. Flexural stiffness of beams and columns ends, K

$$K = \frac{4 \times E_c \times I}{l}$$

Where K is referred to as stiffness factor at beam or column end and can be defined as the amount of moment required to rotate the end of the beam or column 1 rad.

$$I = \frac{b \times h^3}{12}$$

$$E_c = (3,300\sqrt{f_c'} + 6,900)\left(\frac{\gamma_c}{2,300}\right)^{1.5} = (3,300\sqrt{25} + 6,900)\left(\frac{2,447}{2,300}\right)^{1.5} = 25,684 \text{ MPa}$$ ***CSA A23.3-14(8.6.2.2)***

For member AB:

$l = 7.5$ m

$$I = \frac{300 \times 400^3}{12} = 1.6 \times 10^9 \text{ mm}^2$$

$$E = (3,300\sqrt{25} + 6,900)\left(\frac{2,447}{2,300}\right)^{1.5} = 25,684 \text{ MPa}$$

$$K_{AB} = \frac{4 \times 25,684 \times 1.6 \times 10^9}{7500} = 2.19 \times 10^{10} \text{ N.mm}$$

1.3. Distribution factor, DF

$$DF = \frac{K}{\sum K}$$

The distribution factor for a member that is connected to a fixed joint is defined as the fraction of the total resisting moment supplied by this member.

For member AB:

$$DF_{AB}=\frac{2.19\times10^{10}}{2.19\times10^{10}+7.31\times10^{10}+7.31\times10^{10}}=0.130$$

1.4. Flexural stiffness of beams and columns ends, COF

COF = 0.5

Where COF is the Carry-Over Factor that represents the fraction of the moment that is “carried over” from the joint to the beam end when the beam far end is fixed.

1.5. Fixed-end moments, FEMs

For a beam with uniformly distributed load and fixed ends, *FEM* can be found using the following equation:

$$FEM=\frac{w\times l^2}{12}$$

For member AB for load pattern I:

$$FEM_{AB}=\frac{31.75\times7.5^2}{12}=148.83\text{ kN.m}$$

1.6. Beam analysis using moment distribution method

Repeat the previous steps to all frame members to obtain the parameters necessary for the analysis. Moment distribution for the five loading conditions is shown in Table 1. Counter-clockwise rotational moments acting on member ends are taken as positive. Positive span moments are determined from the following equation:

$$M_{u(midspan)}=M_o-\frac{(M_{uL}+M_{uR})}{2}$$

Where M_o is the moment at the midspan for a simple beam, M_{uL} and M_{uR} are the negative moment at the span left and right end, respectively.

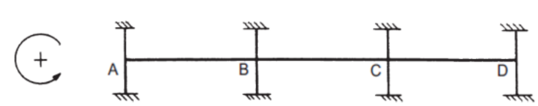
When the end moments are not equal, the maximum moment in the span does not occur at the midspan, but its value is close to that midspan for this example.

Positive moment in span A-B for load pattern I:

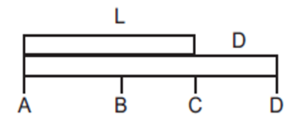
$$M_u^+=\frac{31.75\times7.5^2}{8}-\frac{(134.4+147.4)}{2}=82.3\text{ kN.m}$$

Table 1 – Moment Distribution‡

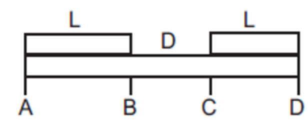
Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	0.130	0.107	0.179	0.174	0.130	0.158
COF	0.500	0.500	0.500	0.500	0.500	0.500



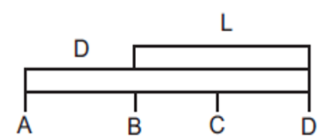
Load Pattern I (S2*)						
FEM	148.8	-148.8	53.6	-53.6	63.8	-63.8
Dist	-19.4	10.2	17.0	-1.8	-1.3	10.1
CO	5.1	-9.7	-0.9	8.5	5.0	-0.7
Dist	-0.7	1.1	1.9	-2.4	-1.8	0.1
CO	0.6	-0.3	-1.2	0.9	0.1	-0.9
Dist	-0.1	0.2	0.3	-0.2	-0.1	0.1
CO	0.1	0.0	-0.1	0.1	0.1	-0.1
Dist	0.0	0.0	0.0	0.0	0.0	0.0
M_u^-	134.4	-147.4	70.6	-48.3	65.7	-55.1
M_u^+	82.3		20.9		35.3	



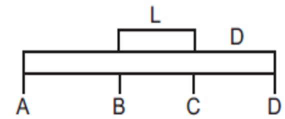
Load Pattern II (Odd*)						
FEM	148.8	-148.8	35.9	-35.9	95.3	-95.3
Dist	-19.4	12.1	20.2	-10.3	-7.8	15.0
CO	6.1	-9.7	-5.2	10.1	7.5	-3.9
Dist	-0.8	1.6	2.7	-3.1	-2.3	0.6
CO	0.8	-0.4	-1.5	1.3	0.3	-1.2
Dist	-0.1	0.2	0.3	-0.3	-0.2	0.2
CO	0.1	-0.1	-0.1	0.2	0.1	-0.1
Dist	0.0	0.0	0.0	0.0	0.0	0.0
M_u^-	135.5	-145.1	52.2	-38.0	92.9	-84.5
M_u^+	83.0		8.7		54.2	



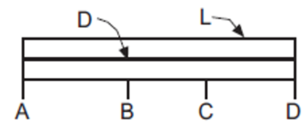
Load Pattern III (S3*)						
FEM	99.6	-99.6	53.6	-53.6	95.3	-95.3
Dist	-13.0	4.9	8.2	-7.3	-5.4	15.0
CO	2.5	-6.5	-3.6	4.1	7.5	-2.7
Dist	-0.3	1.1	1.8	-2.0	-1.5	0.4
CO	0.5	-0.2	-1.0	0.9	0.2	-0.8
Dist	-0.1	0.1	0.2	-0.2	-0.2	0.1
CO	0.1	0.0	-0.1	0.1	0.1	-0.1
Dist	0.0	0.0	0.0	0.0	0.0	0.0
M_u^-	89.3	-100.2	59.1	-57.9	95.9	-83.2
M_u^+	54.7		21.9		53.3	



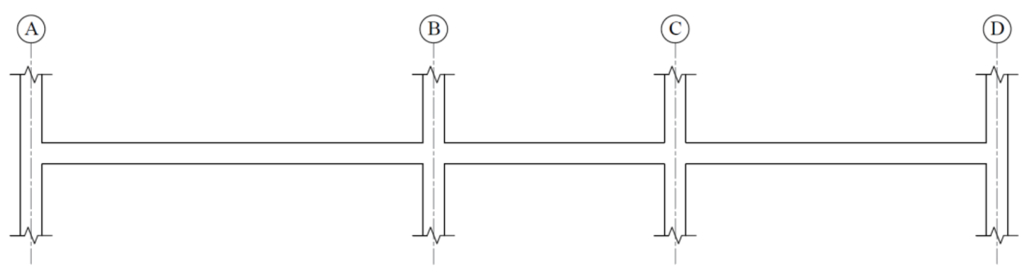
Load Pattern IV (Even*)						
FEM	99.6	-99.6	53.6	-53.6	63.8	-63.8
Dist	-13.0	4.9	8.2	-1.8	-1.3	10.1
CO	2.5	-6.5	-0.9	4.1	5.0	-0.7
Dist	-0.3	0.8	1.3	-1.6	-1.2	0.1
CO	0.4	-0.2	-0.8	0.7	0.1	-0.6
Dist	-0.1	0.1	0.2	-0.1	-0.1	0.1
CO	0.1	0.0	-0.1	0.1	0.0	0.0
Dist	0.0	0.0	0.0	0.0	0.0	0.0
M_u^-	89.2	-100.5	61.6	-52.2	66.3	-54.8
M_u^+	58.1		23.5		37.4	



Load Pattern V (All*)						
FEM	148.8	-148.8	53.6	-53.6	95.3	-95.3
Dist	-19.4	10.2	17.0	-7.3	-5.4	15.0
CO	5.1	-9.7	-3.6	8.5	7.5	-2.7
Dist	-0.7	1.4	2.4	-2.8	-2.1	0.4
CO	0.7	-0.3	-1.4	1.2	0.2	-1.0
Dist	-0.1	0.2	0.3	-0.2	-0.2	0.2
CO	0.1	0.0	-0.1	0.2	0.1	-0.1
Dist	0.0	0.0	0.0	0.0	0.0	0.0
M_u^-	134.6	-147.1	68.1	-54.0	95.3	-83.5
M_u^+	82.4		-7.3		53.5	



Envelop of Maximum Moments



Max M_f^- Column Center Line	135.48	-147.40	70.60	-57.94	95.92	-84.55
Max M_f^- Column Face	112.56	-123.87	56.96	-44.34	77.08	-66.41
Max M_f^+ Midspan	82.97		23.48		54.16	

‡ Moments units are kN.m
* Live load pattern designation in spBeam

2. Moment Redistribution

Now that the structural analysis is completed for all applicable live load patterns, and the enveloped moments are determined we can evaluate the impact of moment redistribution.

Redistribution of elastic bending moments can occur prior to failure due to inelastic rotations in regions with high moments. Beam sections having low percentages of tension reinforcement or containing compression reinforcement have low c/d values and so can tolerate larger redistributions. Due to redistribution, the computed bending moments at a support may be reduced provided that the bending moments in each adjacent span are increased to satisfy equilibrium for the loading case under consideration. Since the loading that causes maximum moments in the adjacent spans, accounting for moment redistribution can result in a reduction of the required flexural reinforcement at both the support and span regions. ***CSA A23.3-14 (N9.2.4)***

Except when approximate values for bending moments are used, the negative moments at the supports of continuous flexural members calculated by elastic analysis for any assumed loading arrangement may each be increased or decreased by not more than (30 – 50c/d)%, but not more than 20%, and the modified negative moments shall be used for calculating the moments at sections within the spans. ***CSA A23.3-14 (9.2.4)***

Note that static equilibrium shall be maintained after redistribution of moments for each loading arrangement. The reduced moment shall be used for calculating redistributed moments at all other sections within the spans.

2.1. Reduction percentage calculations

Using d = 355 mm and cover = 30 mm

For negative moment at support D:

First Iteration

Calculate the required reinforcement to resist the negative moment at support D:

$$M_f = 66.41 \text{kN.m}$$

In this example, *jd* is assumed equal to *0.928d*. The assumption will be verified once the area of steel in finalized.

$$jd = 0.928 \times d = 0.928 \times 355 = 329.4 \text{ mm}$$

$$b = 300 \text{ mm}$$

The required reinforcement at initial trial is calculated as follows:

$$A_s = \frac{M_f}{\varphi_s f_y jd} = \frac{66.41 \times 10^6}{0.85 \times 400 \times 329.4} = 592.9 \text{ mm}^2$$

$$\alpha_1 = 0.85 - 0.0015 f_c^{'} = 0.81 > 0.67$$ ***CSA A23.3-14 (10.1.7)***

$$\beta_1 = 0.97 - 0.0025 f_c^{'} = 0.91 > 0.67$$ ***CSA A23.3-14 (10.1.7)***

Recalculate 'a' for the actual A_s = 1,068 mm^2: $a = \frac{\phi_s A_s f_y}{\phi_c \alpha_1 f'_c b} = \frac{0.85 \times 592.9 \times 400}{0.65 \times 0.81 \times 25 \times 400} = 50.9 \text{ mm}$

$$c = \frac{a}{\beta_1} = \frac{50.9}{0.91} = 56 \text{ mm}$$

The tension reinforcement in flexural members shall not be assumed to reach yield unless:

$$\frac{c}{d} \le \frac{700}{700 + f_y}$$ **CSA A23.3-14 (10.5.2)**

$$\frac{56}{355} = 0.16 \le 0.64$$

$$jd = d - a/2 = 0.928\ d$$

Therefore, the assumption that tension reinforcements will yield and *jd* equals to *0.928d* is valid.

$$Adjustment_1 = (30 - 50c/d) = \left(30 - 50 \times \frac{56}{355}\right) = 22.1 > 20 \rightarrow Adjustment_1 = 20\%$$

Second Iteration

$$\left(M_f\right)_2 = 66.41 - 66.41 \times 0.20 = 53.13 \text{ kN.m}$$

jd is assumed equal to *0.944d*.

$$jd = 0.944 \times d = 0.944 \times 355 = 335 \text{ mm}$$

The required reinforcement at initial trial is calculated as follows:

$$A_s = \frac{M_f}{\varphi_s f_y jd} = \frac{53.13 \times 10^6}{0.85 \times 400 \times 335} = 466 \text{ mm}^2$$

Recalculate 'a' for the actual A_s = 466 mm^2: $a = \frac{\phi_s A_s f_y}{\phi_c \alpha_1 f'_c b} = \frac{0.85 \times 466 \times 400}{0.65 \times 0.81 \times 25 \times 400} = 40 \text{ mm}$

$$c = \frac{a}{\beta_1} = \frac{40}{0.91} = 44.1 \text{ mm}$$

The tension reinforcement in flexural members shall not be assumed to reach yield unless:

$$\frac{c}{d} \le \frac{700}{700 + f_y}$$ **CSA A23.3-14 (10.5.2)**

$$\frac{44.1}{355} = 0.12 \le 0.64$$

$$jd = d - a/2 = 0.944\ d$$

Therefore, the assumption that tension reinforcements will yield and *jd* equals to *0.944d* is valid.

$$Adjustment_2 = (30 - 50c/d) = \left(30 - 50 \times \frac{44.1}{355}\right) = 23.8 > 20 \rightarrow Adjustment_2 = 20\%$$

Since $Adjustment_1 = Adjustment_2$ → End of Iterations

Table 2 - Moment Adjustments at Supports							
		Support					
		A	B		C		D
		Right	Left	Right	Left	Right	Left
Iteration 1	M_f, kN.m	112.56	123.87	55.96	44.34	77.08	66.41
	c/d	0.285	0.320	0.131	0.103	0.186	0.158
	Adjustment, %	15.7	14.0	20.0	20.0	20.0	20.0
Iteration 2	M_f, kN.m	94.84	106.52	44.77	35.47	61.67	53.13
	c/d	0.234	0.268	0.104	0.081	0.146	0.124
	Adjustment, %	18.3	16.6	20.0	20.0	20.0	20.0
Iteration 3	M_f, kN.m	91.97	103.30				
	c/d	0.226	0.258				
	Adjustment, %	18.7	17.1				
Iteration 4	M_f, kN.m	91.45	102.60				
	c/d	0.225	0.256				
	Adjustment, %	18.8	17.2				
Iteration 5	M_f, kN.m	91.44					
	c/d	0.225					
	Adjustment, %	18.8					
Final Allowable Adjustment, %		18.8	17.2	20	20	20	20

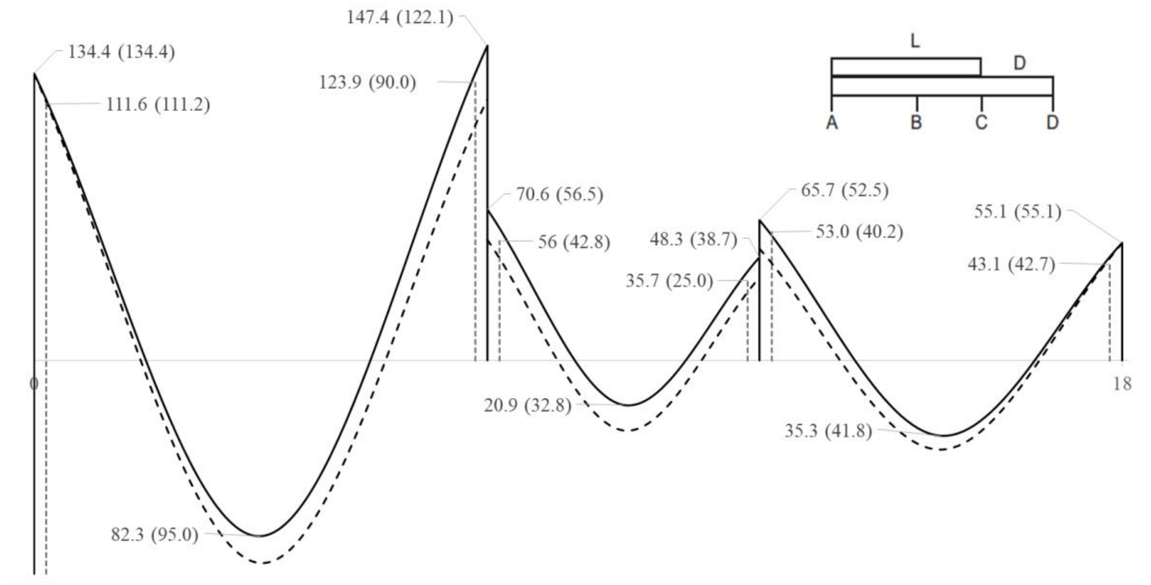
2.2. Adjustment of moments (redistribution)

Now the engineer can make decisions to reduce any negative moments (or positive) based on project parameters including:

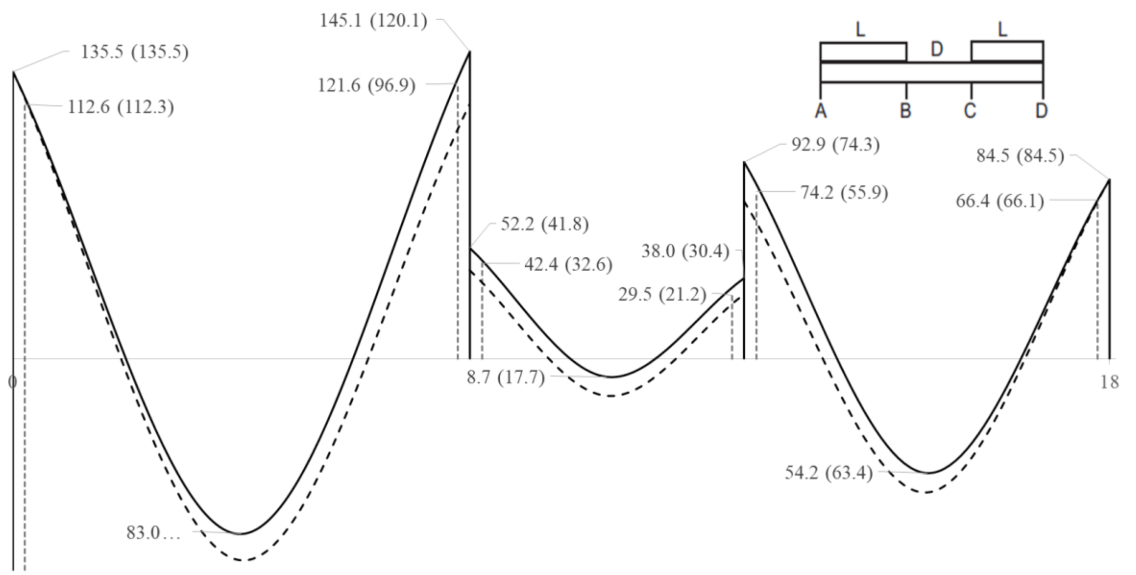
- Steel detailing and placement considerations.
- New design or investigation of existing beams
- Optimize the provided reinforcement for more economical design
- Optimize the provided reinforcement for improved uniformity

It was decided to reduce the negative moments on both sides of supports B and C and accept the increase in the corresponding positive moments, and not to adjust the negative moments at the exterior supports A and D.

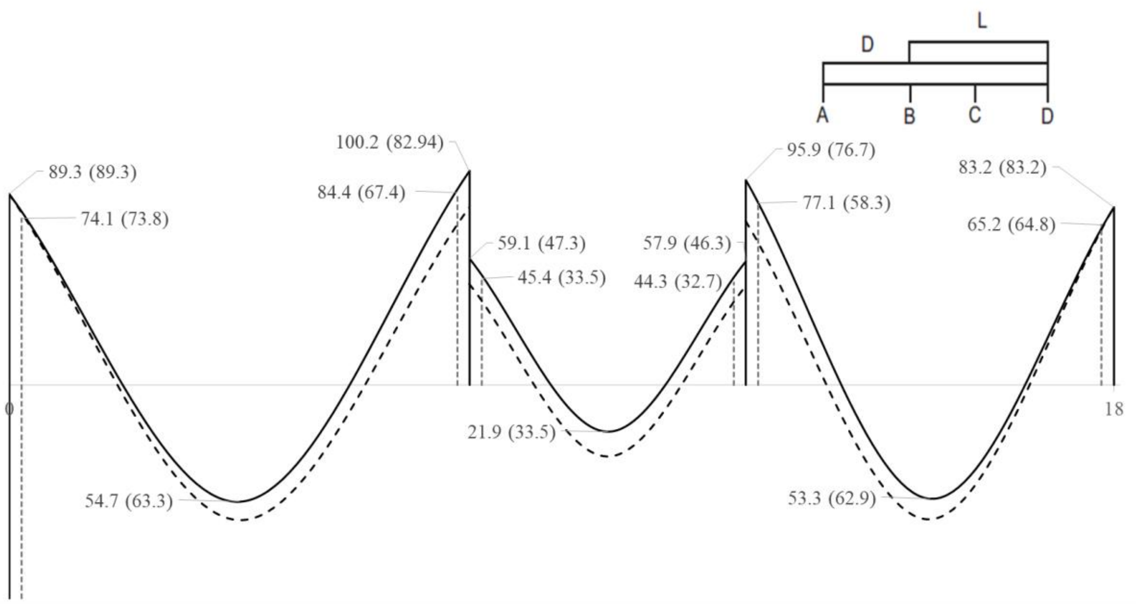
The following figures show the unadjusted and adjusted moment values at the columns centerlines, columns faces, and at the midspan for each load pattern and for the maximum values at each critical location (maximum moment envelops).



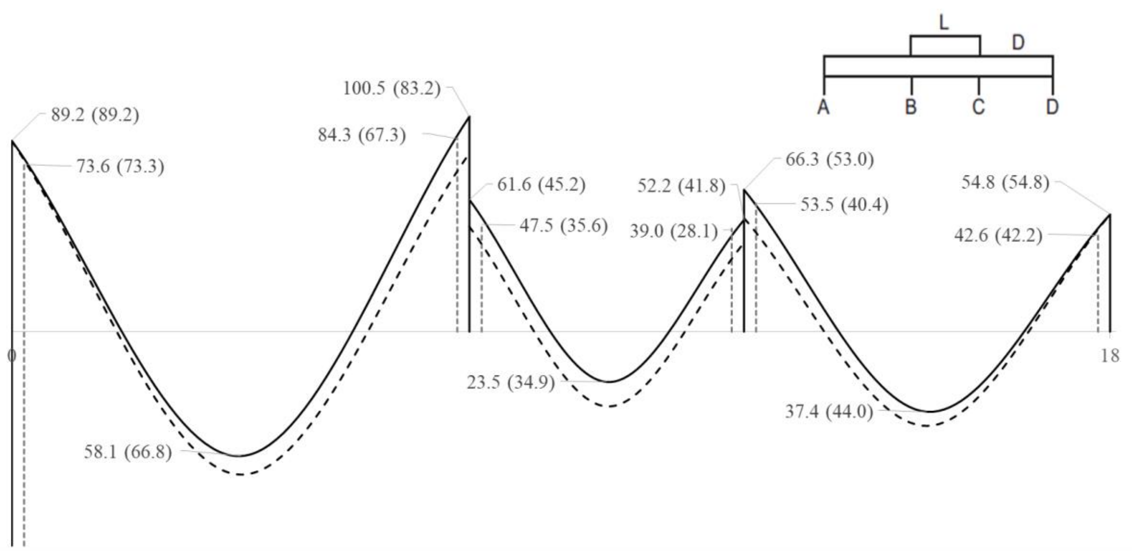
(a) Load Pattern I (moments in kN.m)



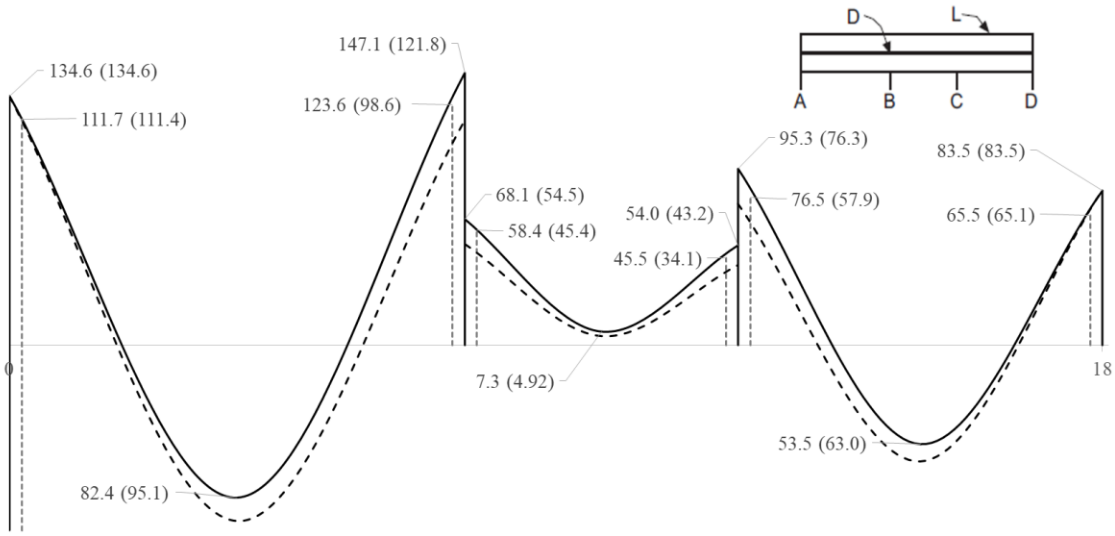
(b) Load Pattern II (moments in kN.m)



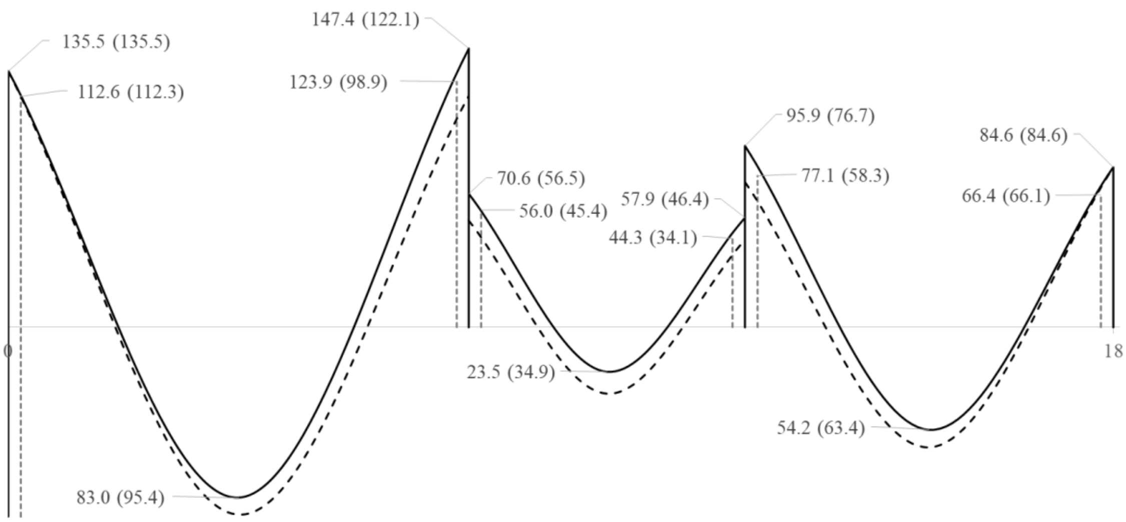
(c) Load Pattern III (moments in kN.m)



(d) Load Pattern IV (moments in kN.m)



(e) Load Pattern V (moments in kN.m)



(f) Maximum Moment Envelopes for Pattern Loading (moments in kN.m)

Figure 2 – Redistribution of Moments (All Load Patterns) (PCA Notes – Example 8-2)

For load pattern I

$M_{B,left} = 147.4$ kN.m (adjustment = 17.2%)

$Adjusted\ M_{B,left} = 147.4 - 147.4 \times 0.172 = 122.0$ kN.m

Increase in positive moment in span A-B

$$M_A = 134.4 \text{ kN.m}$$

$$Adjusted\ M_{B,left} = 122.0 \text{ kN.m}$$

$$Moment\ due\ to\ uniform\ load = \frac{w_f \times l^2}{8} = \frac{31.75 \times 7.5^2}{8} = 223.2 \text{ kN.m}$$

$$Adjusted\ \ positive\ moment\ at\ mid\text{-}span = 223.2 - \frac{134.4 + 122.0}{2} = 95.0 \text{ kN.m}$$

Decrease in negative moment at the left face of support B

$$Ordinate\ on\ line\ M_A\ to\ M_{B,left} = 134.4 + \frac{122.1 - 134.4}{7.5} \times 7.3 = 122.3 \text{ kN}$$

$$Moment\ due\ to\ uniform\ load = \frac{1}{2} \times w_f \times x \times (l - x) = \frac{1}{2} \times 31.75 \times 7.30 \times (7.5 - 7.3) = 23.3 \text{ kN.m}$$

$$Adjusted\ negative\ moment\ at\ the\ left\ \ face\ of\ \ support\ B = 122.3 - 23.3 = 99.0 \text{ kN.m}$$

Similar calculations are made to determine the adjusted moment at other locations and for other load patterns. Results of the additional calculations are shown in the following table.

Table 3 - Moments Before and After Redistribution (Moments in kN.m)										
Location	Load Pattern I S2		Load Pattern II Odd		Load Pattern III S3		Load Pattern IV Even		Load Pattern V all	
	M_f	M_{adj}	M_f	M_{adj}	M_f	M_{adj}	M_f	M_{adj}	M_f	M_{adj}
A Center	-134.4	-134.4	-135.5	-135.5	-89.3	-89.3	-89.2	-89.2	-134.6	-134.6
A Right Face	-111.6	-111.2	-112.6	-112.3	-74.1	-73.8	-73.6	-73.3	-111.7	-111.4
Midspan A-B	82.3	95	83	95.4	54.7	63.3	58.1	66.8	82.4	95.1
B Left Face	-123.9	-98.9	-121.6	-96.9	-84.4	-67.4	-84.3	-67.3	-123.6	-98.6
B Left Center	-147.4	-112	-145.1	-120.1	-100.2	-82.9	-100.5	-83.2	-147.1	-121.8
B Right Center	-70.6	-56.5	-52.2	-41.8	-59.1	-47.3	-61.6	-49.2	-68.1	-54.5
B Right Face	-56	-42.8	-42.2	-32.6	-45.4	-33.6	-47.5	-35.6	-58.4	-45.4
Midspan B-C	20.9	32.8	8.7	17.7	21.9	33.5	23.5	34.9	-7.3	4.9
C Left Face	-35.7	-25	-29.5	-21.2	-44.3	-32.7	-39	-28.1	-45.5	-34.1
C Left Center	-48.3	-38.7	-38	-30.4	-57.9	-46.35	-52.2	-41.8	-54	-43.2
C Right Center	-65.7	-52.5	-92.9	-74.3	-95.9	-76.7	-66.3	-53	-95.3	-76.3
C Right Face	-53	-40.2	-74.2	-55.9	-77.1	-58.3	-53.3	-40.4	-76.5	-57.9
Midspan C-D	35.3	41.8	54.2	63.4	53.3	62.9	37.4	44	53.5	-63
D Left Face	-43.1	-42.7	-66.4	-66.1	-65.2	-64.8	-42.6	-42.2	-65.5	-65.1
D Center	-55.1	-55.1	-84.5	-84.5	-83.2	-83.2	-54.8	-54.8	-83.5	-83.5

(shaded) Final design moments after redistribution for critical sections (left and right support face & midspan)

After the adjusted bending moments have been determined analytically, the adjusted bending moment diagrams for each load pattern can be determined. The adjusted moment curves were determined graphically and are indicated by the dashed lines in Figures 2 (a) through (e).

An Adjusted maximum moment envelope can now be obtained from the adjusted moment curves as shown in Figure 2 (f) by dashed lines.

From the redistribution moment envelopes of Figure 2 (f), the design factored moments and the required reinforcement area are obtained as shown in following table. Check example "One-Way Wide Module (Skip) Joist Concrete Floor System Design" for detailed calculations for flexural and shear design of continuous beams.

Table 4 - Summary of Final Design (comparison of % reduction and required Reinforcement)

Location		Moment at column face, kN.m		Load Case	A_s, mm^2		
		Undistributed	Redistributed		Undistributed	Redistributed	%
Support	Right	-112.6	-112.3	II	1070	1068	99.8
Midspan A-B		83.0	95.4	II	756	885	117.0
Support B	Left	-123.9	-98.9	I	1200	923	76.9
	Right	-58.4	-45.4	V	516	395	76.6
Midspan B-C		23.5	34.9	IV	*300*	*300*	100
Support C	Left	-45.5	-34.1	V	396	*300*	75.8
	Right	-77.1	-58.3	III	697	515	73.9
Midspan C-D		54.2	63.4	II	476	564	118.3
Support	Left	-66.4	-66.1	II	593	589	99.4

Italic underlined values indicate $A_{s,min}$ governs

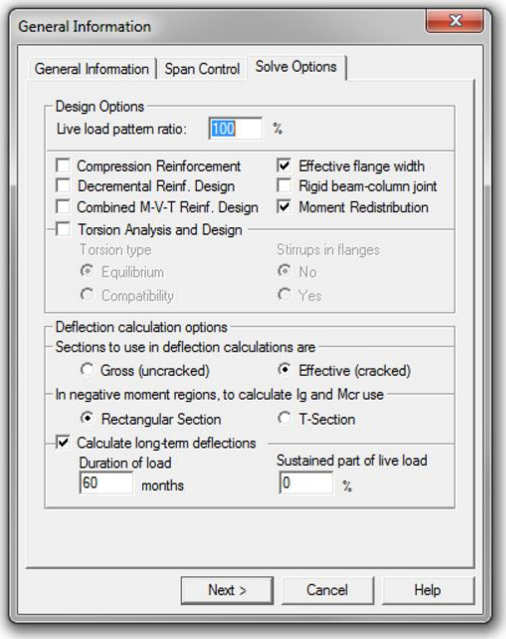
Where $A_{s,\min} = \dfrac{0.2 \times \sqrt{f_c^{'}}}{f_y} \times b_t \times h = \dfrac{0.2\sqrt{25}}{400} \times 400 \times 300 = 300\ \text{mm}^2$ ***CSA A23.3-14 (10.5.1.2)***

3. Continuous Beam Analysis and Design Using Moment Redistribution – spBeam Software

spBeam is widely used for analysis, design and investigation of beams, and one-way slab systems (including standard and wide module joist systems) per latest Canadian (CSA A23.3-14) and American (ACI 318-14) codes. spBeam can be used for new designs or investigation of existing structural members subjected to flexure, shear, and torsion loads. With capacity to integrate up to 20 spans and two cantilevers of wide variety of floor system types, spBeam is equipped to provide cost-effective, accurate, and fast solutions to engineering challenges.

spBeam provides top and bottom bar details including development lengths and material quantities, as well as live load patterning and immediate and long-term deflection results. Using the moment redistribution feature engineers can deliver safe designs with savings in materials and labor. Engaging this feature allows up to 20% reduction of negative moments over supports reducing reinforcement congestions in these areas.

Redistribution of negative moments applies to one-way and beam systems only. It can be engaged using the "*Input Redistribution"* option on the *"Solve Options"* tab in the *"General Information"* dialog box (see the following figure). The program allows for redistribution of negative moments at supports. Only reduction in negative moments is considered. Increase of negative moments at the support is not taken into account even though it is allowed by the code. Static equilibrium is maintained meaning that bending moments and shear forces along the span are adjusted in accordance with the reduction of moments applied at the supports.



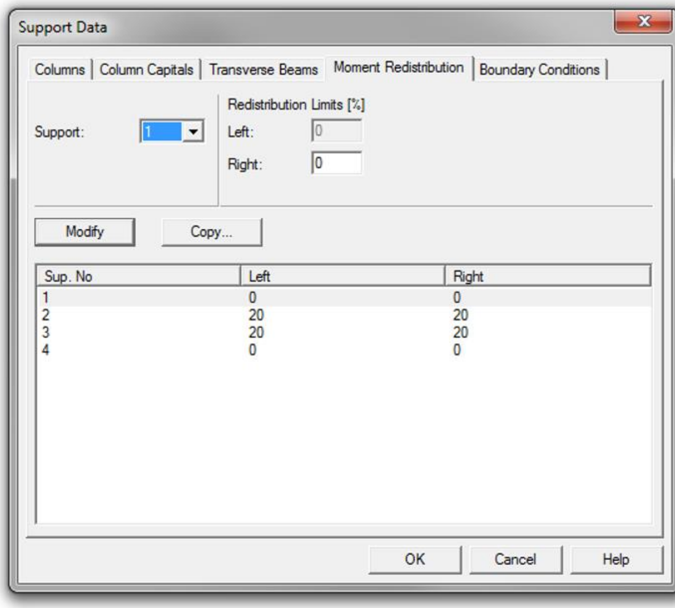


Figure 3 –Activating Moment Redistribution (spBeam)

From elastic static analysis, the largest moments from all load combinations and load patterns are determined at support faces on both ends of each span except cantilevers. These moments are used to calculate the maximum code allowable adjustment percentage of calculated moment.

In the investigation mode, program uses the area of provided reinforcement to obtain redistribution factors. In the design mode the required reinforcement area is used. The reduction percentage is limited to 20% and not to exceed the maximum values specified by the user. Negative moments at span ends are reduced by the amount of redistribution factors and new moment values are iteratively used to obtain new redistribution factors. This iterative procedure is repeated until the change in distribution factor is negligible (does not exceed 0.01%), but no more than 10 times.

For illustration and comparison purposes, the following figures provide a sample of the results obtained from an spBeam model created for the continuous beam in this example. Special emphasis can be given to Figure 7 that illustrated the maximum and adjusted moments for span 2.

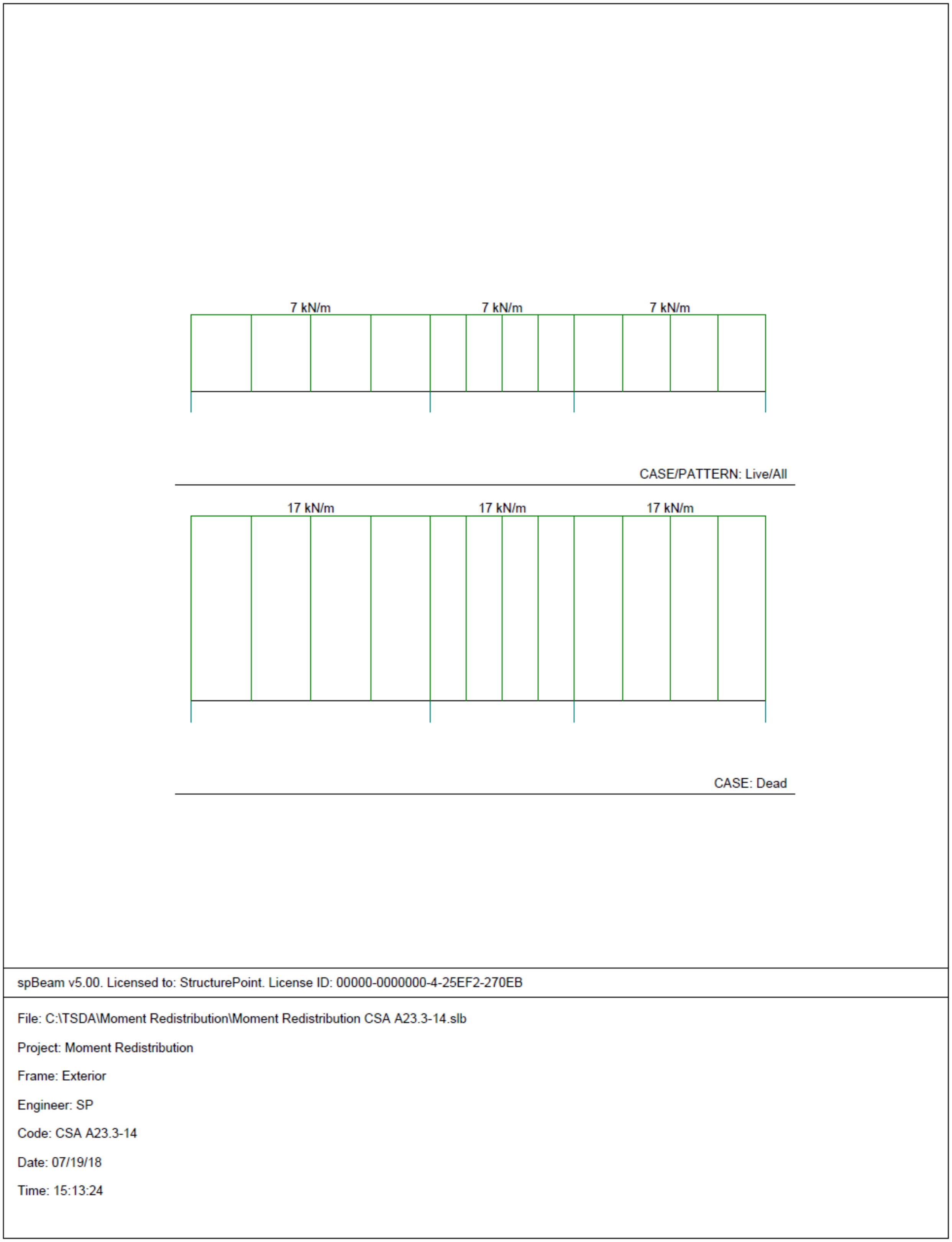
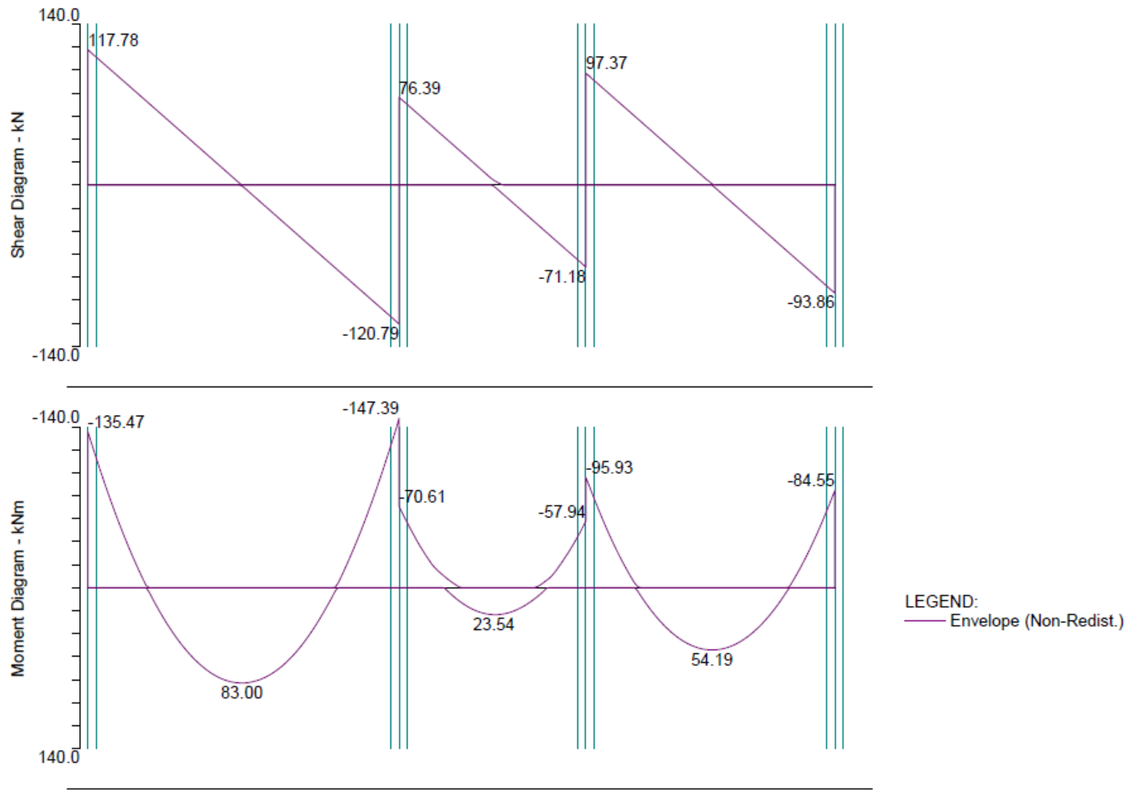


Figure 4 – Loading (spBeam)



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File: C:\TSDA\Moment Redistribution\Moment Redistribution CSA A23.3-14.slb

Project: Moment Redistribution

Frame: Exterior

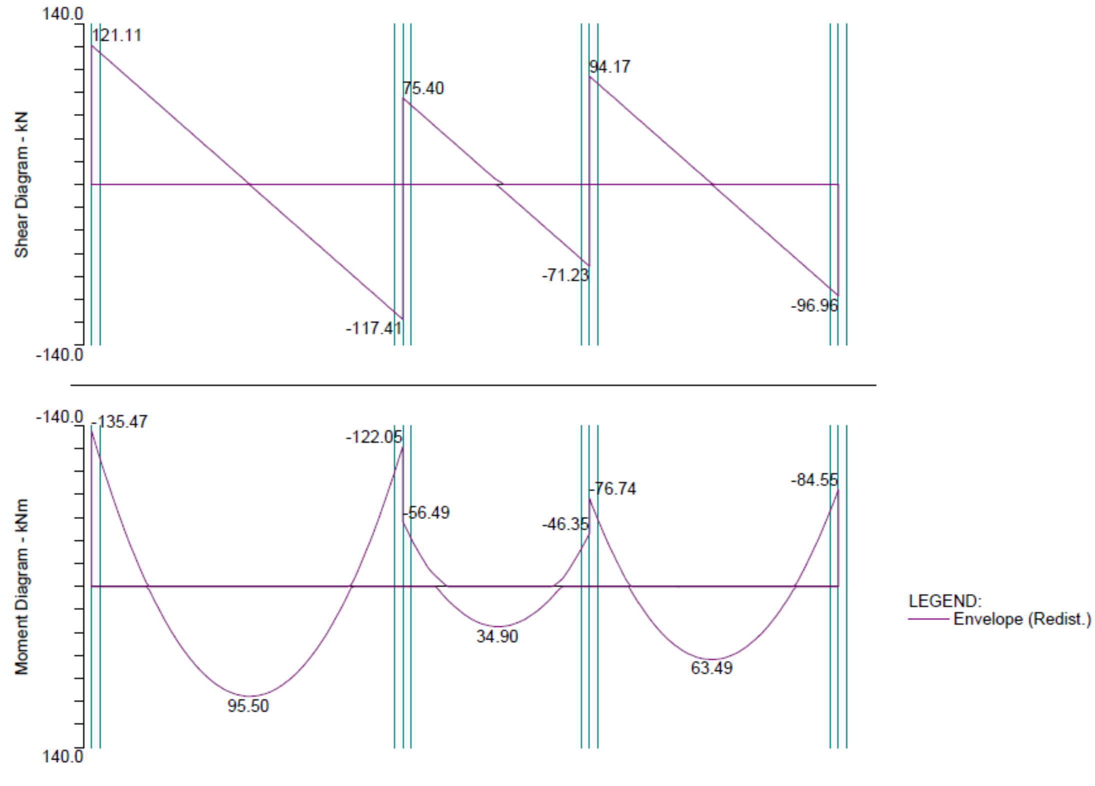
Engineer: SP

Code: CSA A23.3-14

Date: 07/19/18

Time: 15:15:44

Figure 5 – Internal Forces before Moment Redistribution (spBeam)



spBeam v5.00. Licensed to: StructurePoint. License ID: 00000-0000000-4-25EF2-270EB

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Project: Moment Redistribution

Frame: Exterior

Engineer: SP

Code: CSA A23.3-14

Date: 07/19/18

Time: 15:16:30

Figure 6 – Internal Forces after Moment Redistribution (spBeam)

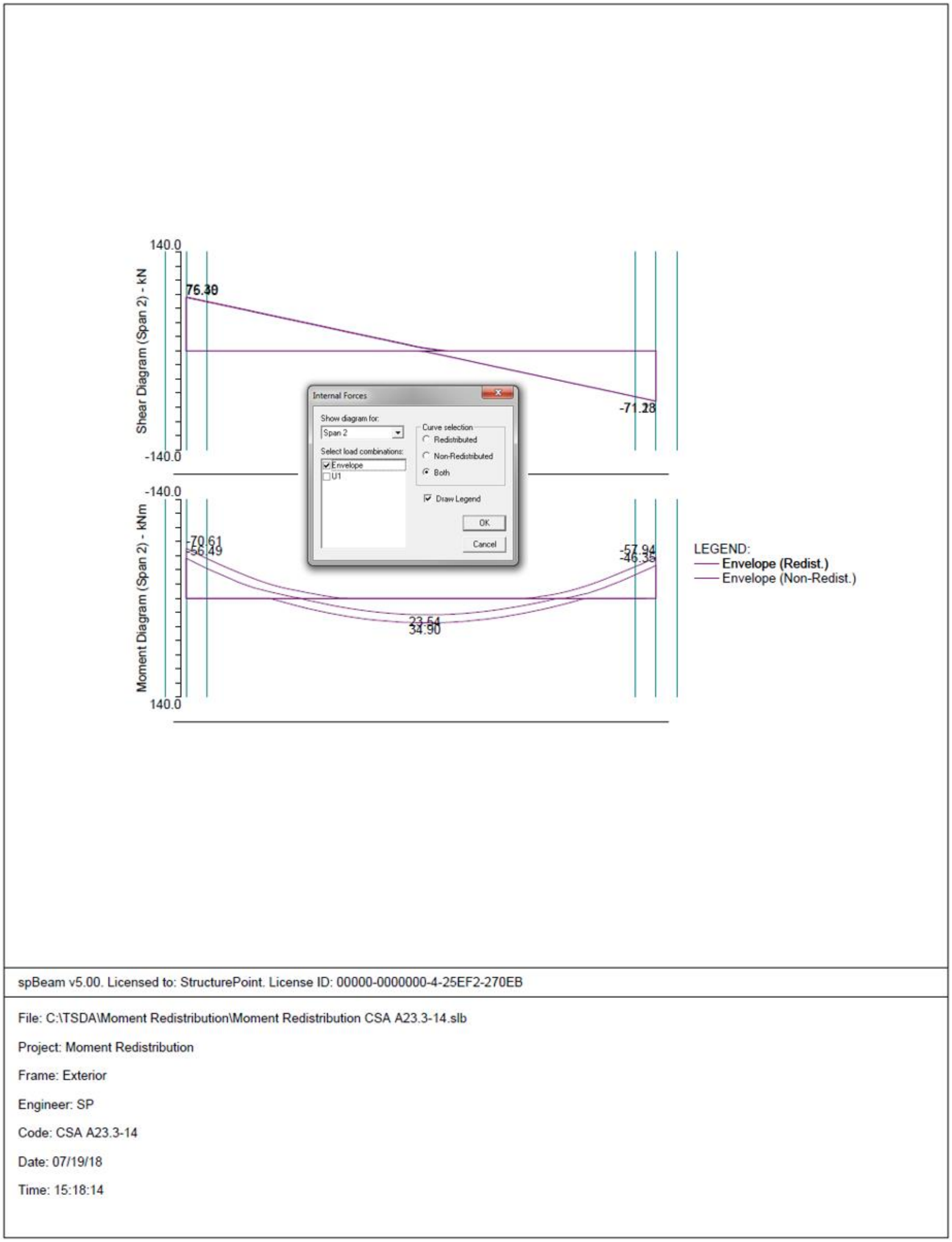
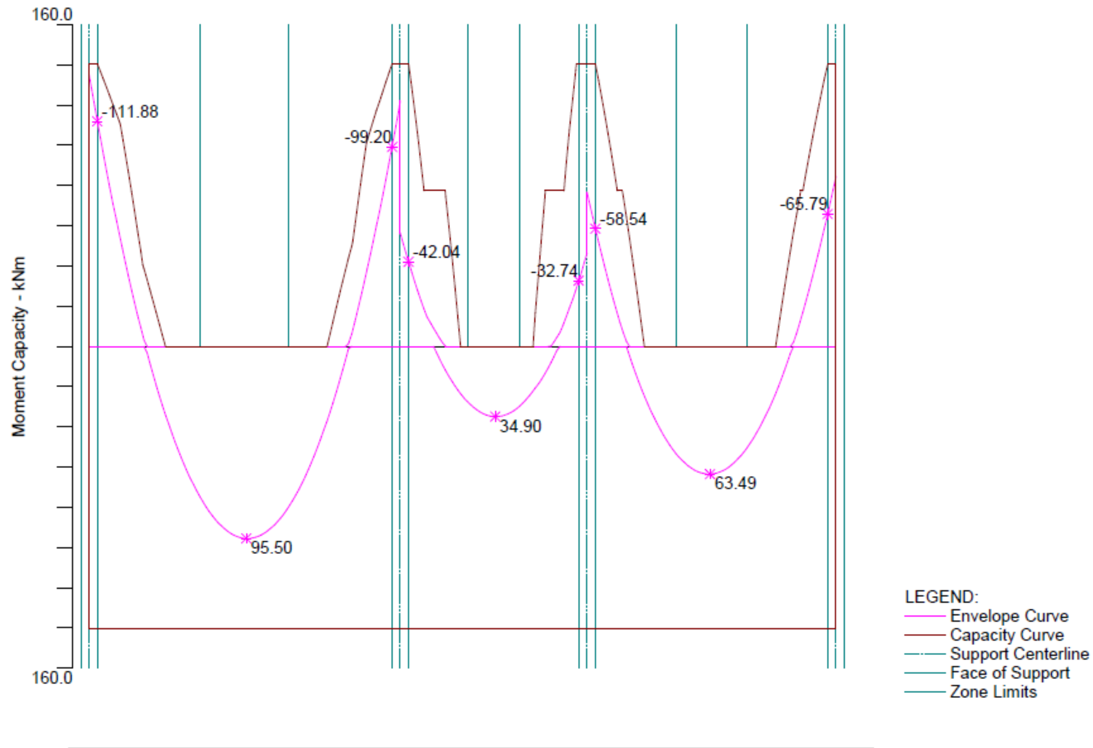


Figure 7 – Internal Forces before and after Moment Redistribution for Span 2 (spBeam)



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Project: Moment Redistribution

Frame: Exterior

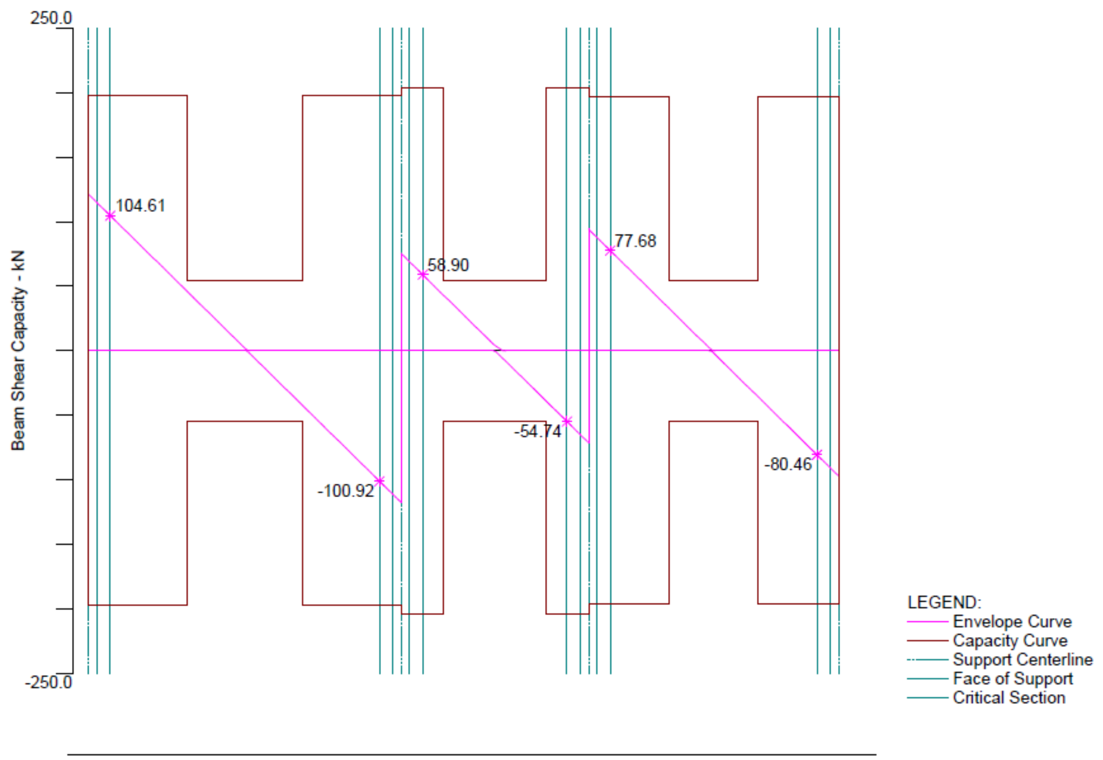
Engineer: SP

Code: CSA A23.3-14

Date: 07/19/18

Time: 15:19:55

Figure 8 – Moment Capacity Diagram after Moment Redistribution (spBeam)



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Project: Moment Redistribution

Frame: Exterior

Engineer: SP

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Date: 07/19/18

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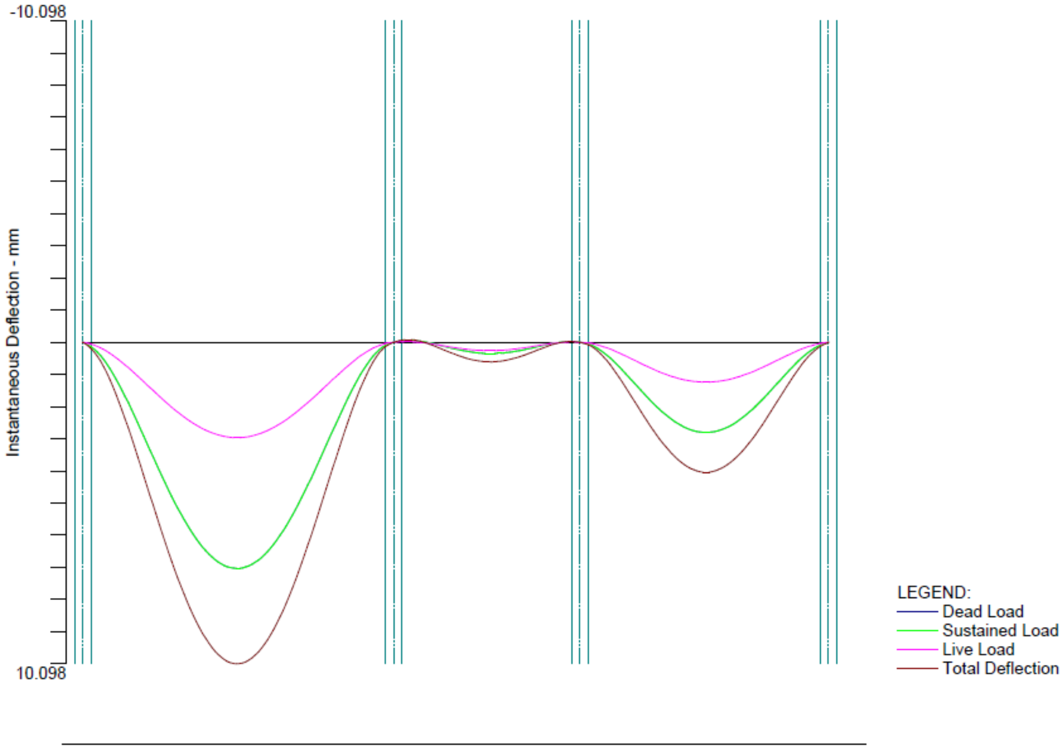
Figure 9 – Shear Capacity Diagram after Moment Redistribution (spBeam)



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Project: Moment Redistribution

Frame: Exterior

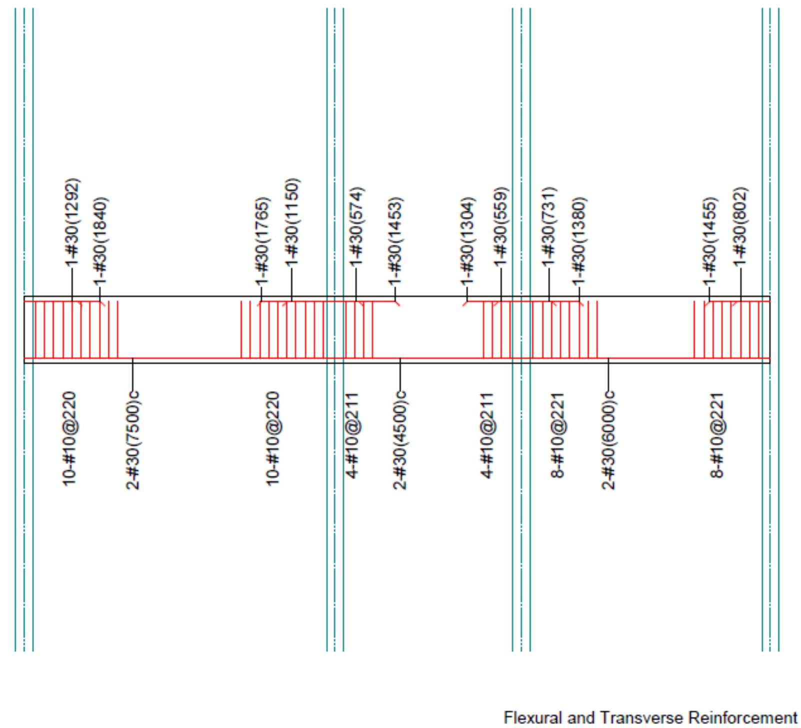
Engineer: SP

Code: CSA A23.3-14

Date: 07/19/18

Time: 15:21:00

Figure 10 – Immediate Deflection Diagram (spBeam)



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Project: Moment Redistribution

Frame: Exterior

Engineer: SP

Code: CSA A23.3-14

Date: 07/19/18

Time: 15:21:25

Figure 11 – Reinforcement after Moment Redistribution (spBeam)

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      =================================================================================================
                                        spBeam v5.00 (TM)
                      A Computer Program for Analysis, Design, and Investigation of
                           Reinforced Concrete Beams and One-way Slab Systems
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      program.

==================================================================================================================
[1] INPUT ECHO
==================================================================================================================

General Information
===================
   File name: C:\TSDA\Moment Redistribution\Moment Redistribution CSA A23.3-14.slb
   Project: Moment Redistribution
   Frame: Exterior
   Engineer: SP
   Code: CSA A23.3-14
   Reinforcement Database: CSA G30.18
   Mode: Design
   Number of supports = 4
   Floor System: One-Way/Beam

   Live load pattern ratio = 100%
   Deflections are based on cracked section properties.
   In negative moment regions, Ig and Mcr DO NOT include flange/slab contribution (if available)
   Long-term deflections are calculated for load duration of 60 months.
   0% of live load is sustained.
   Compression reinforcement calculations NOT selected.
   Default incremental rebar design selected.
   Combined M-V-T reinforcement design NOT selected.
   Moment redistribution selected.
   Effective flange width calculations selected.
   Rigid beam-column joint NOT selected.
   Torsion analysis and design NOT selected.

Material Properties
===================
             Slabs|Beams        Columns
            ------------   ------------
   wc    =        2447.3         2447.3 kg/m3
   f'c   =            25             25 MPa
   Ec    =         25684          25684 MPa
   fr    =           1.5            1.5 MPa
   Precast concrete construction is not selected.

   fy    =           400 MPa, Bars are not epoxy-coated
   fyt   =           400 MPa
   Es    =        200000 MPa

Reinforcement Database
======================
   Units: Db (mm), Ab (mm^2), Wb (kg/m)
   Size       Db       Ab       Wb     Size       Db       Ab       Wb
   ---- -------- -------- --------     ---- -------- -------- --------
    #10       11      100        1      #15       16      200        2
    #20       20      300        2      #25       25      500        4
    #30       30      700        5      #35       36     1000        8
    #45       44     1500       12      #55       56     2500       20

Span Data
=========
```

Slabs

Units: L1, wL, wR (m); t, bEff, Hmin (mm)

Span	Loc	L1	t	wL	wR	bEff	Hmin
1	ExtL	7.500	0	0.150	0.150	300	0
2	ExtL	4.500	0	0.150	0.150	300	0
3	ExtL	6.000	0	0.150	0.150	300	0

Ribs and Longitudinal Beams

Units: b, h, Sp (mm)

	Ribs			Beams		Span
Span	b	h	Sp	b	h	Hmin
1	0	0	0	300	400	394
2	0	0	0	300	400	195
3	0	0	0	300	400	311

Support Data

Columns

Units: c1a, c2a, c1b, c2b (mm); Ha, Hb (m)

Supp	c1a	c2a	Ha	c1b	c2b	Hb	Red%
1	400	400	3.000	400	400	3.000	100
2	400	400	3.000	400	400	3.000	100
3	400	400	3.000	400	400	3.000	100
4	400	400	3.000	400	400	3.000	100

Moment Redistribution Limits

Supp	Left[%]	Right[%]
1	0	0
2	20	20
3	20	20
4	0	0

Boundary Conditions

Units: Kz (kN/mm); Kry (kN-mm/rad)

Supp	Spring Kz	Spring Kry	Far End A	Far End B
1	0	0	Fixed	Fixed
2	0	0	Fixed	Fixed
3	0	0	Fixed	Fixed
4	0	0	Fixed	Fixed

Load Data

Load Cases and Combinations

Case	Dead	Live
Type	DEAD	LIVE
U1	1.250	1.500

Line Loads

Units: Wa, Wb (kN/m), La, Lb (m)

Case/Patt	Span	Wa	La	Wb	Lb
Dead	1	17.00	0.000	17.00	7.500
	2	17.00	0.000	17.00	4.500
	3	17.00	0.000	17.00	6.000
Live	1	7.00	0.000	7.00	7.500
	2	7.00	0.000	7.00	4.500
	3	7.00	0.000	7.00	6.000
Live/Odd	1	7.00	0.000	7.00	7.500
	3	7.00	0.000	7.00	6.000
Live/Even	2	7.00	0.000	7.00	4.500
Live/S1	1	7.00	0.000	7.00	7.500
Live/S2	1	7.00	0.000	7.00	7.500
	2	7.00	0.000	7.00	4.500
Live/S3	2	7.00	0.000	7.00	4.500
	3	7.00	0.000	7.00	6.000
Live/S4	3	7.00	0.000	7.00	6.000

Reinforcement Criteria

Slabs and Ribs

	Top bars		Bottom bars	
	Min	Max	Min	Max
Bar Size	#35	#35	#35	#35

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     Bar spacing      25       457       25       457 mm
     Reinf ratio    0.14      5.00     0.14      5.00 %
     Cover            38                 38           mm
     There is NOT more than 300 mm of concrete below top bars.

   Beams
   -----
                  _____Top bars___  __Bottom bars___   ____Stirrups____
                     Min      Max      Min      Max       Min      Max
                  -------  -------  -------  -------   -------  -------
     Bar Size        #30      #30      #30      #30       #10      #10
     Bar spacing      25      457       25      457       152      457 mm
     Reinf ratio    0.14     5.00     0.14     5.00 %
     Cover            30                30          mm
     Layer dist.      25                25          mm
     No. of legs                                            2        6
     Side cover                                            30          mm
     1st Stirrup                                           76          mm
     There is NOT more than 300 mm of concrete below top bars.
```

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Reinforced Concrete Beams and One-way Slab Systems

[2] DESIGN RESULTS

Moment Redistribution Factors

Units: Org.Mu (kNm)

			Calculated			User	Applied
Supp	Side	Org.Mu	Iter.#	c/d	Factor[%]	Limit[%]	Factor[%]
1	Right	112.55	5	0.22471	18.76	0.00	0.00
2	Left	123.87	6	0.25614	17.19	20.00	17.19
2	Right	55.96	2	0.10368	20.00	20.00	20.00
3	Left	44.34	2	0.08128	20.00	20.00	20.00
3	Right	77.09	2	0.14574	20.00	20.00	20.00
4	Left	66.41	2	0.12425	20.00	0.00	0.00

Top Reinforcement

Units: Width (m), Mmax (kNm), Xmax (m), As (mm^2), Sp (mm)

Span	Zone	Width	Mmax	Xmax	AsMin	AsMax	AsReq	SpProv	Bars
1	Left	0.30	111.88	0.200	300	2389	1064	183	2-#30
	Midspan	0.30	0.00	3.750	0	2389	0	0	---
	Right	0.30	99.20	7.300	300	2389	925	183	2-#30
2	Left	0.30	42.04	0.200	300	2389	364	183	2-#30
	Midspan	0.30	0.00	2.250	0	2389	0	0	---
	Right	0.30	32.74	4.300	300	2389	281	183	2-#30 *3
3	Left	0.30	58.54	0.200	300	2389	517	183	2-#30
	Midspan	0.30	0.00	3.000	0	2389	0	0	---
	Right	0.30	65.79	5.800	300	2389	587	183	2-#30

NOTES:
*3 - Design governed by minimum reinforcement.

Top Bar Details

Units: Length (m)

	Left				Continuous		Right			
Span	Bars	Length	Bars	Length	Bars	Length	Bars	Length	Bars	Length
1	1-#30	1.84	1-#30*	1.29	---		1-#30	1.76	1-#30*	1.15
2	1-#30	1.45	1-#30*	0.57	---		1-#30	1.30	1-#30*	0.56
3	1-#30	1.38	1-#30*	0.73	---		1-#30	1.45	1-#30*	0.80

NOTES:
* - Bar cut-off location shall be manually checked for compliance with CSA A23.3, 11.2.13.

Top Bar Development Lengths

Units: Length (mm)

	Left				Continuous		Right			
Span	Bars	Length	Bars	DevLen	Bars	DevLen	Bars	DevLen	Bars	DevLen

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1	1-#30	1091.77	1-#30	1091.77	---		1-#30	949.80	1-#30	949.80
2	1-#30	373.95	1-#30	373.95	---		1-#30	300.00	1-#30	300.00
3	1-#30	531.03	1-#30	531.03	---		1-#30	602.12	1-#30	602.12

Bottom Reinforcement
====================

Units: Width (m), Mmax (kNm), Xmax (m), As (mm^2), Sp (mm)

Span	Width	Mmax	Xmax	AsMin	AsMax	AsReq	SpProv	Bars
1	0.30	95.50	3.787	300	2389	886	183	2-#30
2	0.30	34.90	2.287	300	2389	300	183	2-#30 *3
3	0.30	63.49	2.963	300	2389	564	183	2-#30

NOTES:
*3 - Design governed by minimum reinforcement.

Bottom Bar Details
==================

Units: Start (m), Length (m)

	Long Bars			Short Bars		
Span	Bars	Start	Length	Bars	Start	Length
1	2-#30	0.00	7.50	---		
2	2-#30	0.00	4.50	---		
3	2-#30	0.00	6.00	---		

Bottom Bar Development Lengths
==============================

Units: DevLen (mm)

	Long Bars		Short Bars	
Span	Bars	DevLen	Bars	DevLen
1	2-#30	909.49	---	
2	2-#30	307.91	---	
3	2-#30	579.43	---	

Flexural Capacity
=================

Units: x (m), As (mm^2), PhiMn, Mu (kNm)

		Top						Bottom					
Span	x	AsTop	PhiMn-	Mu-	Comb	Pat	Status	AsBot	PhiMn+	Mu+	Comb	Pat	Status
1	0.000	1400	-140.40	-135.47	U1	Odd	---	1400	140.40	0.00	U1	All	---
	0.200	1400	-140.40	-111.88	U1	Odd	OK	1400	140.40	0.00	U1	All	OK
	0.748	1049	-110.56	-53.81	U1	Odd	OK	1400	140.40	0.00	U1	All	OK
	1.292	351	-40.60	-5.54	U1	Odd	OK	1400	140.40	0.00	U1	All	OK
	1.840	0	0.00	0.00	U1	All	OK	1400	140.40	33.91	U1	S2	OK
	2.685	0	0.00	0.00	U1	All	OK	1400	140.40	75.24	U1	Odd	OK
	3.750	0	0.00	0.00	U1	All	OK	1400	140.40	95.42	U1	Odd	OK
	3.787	0	0.00	0.00	U1	All	OK	1400	140.40	95.50	U1	Odd	OK
	4.815	0	0.00	0.00	U1	All	OK	1400	140.40	79.60	U1	Odd	OK
	5.735	0	0.00	0.00	U1	All	OK	1400	140.40	36.94	U1	Odd	OK
	6.350	453	-51.72	-8.06	U1	S2	OK	1400	140.40	0.00	U1	All	OK
	6.685	947	-101.21	-36.92	U1	S2	OK	1400	140.40	0.00	U1	All	OK
	7.300	1400	-140.40	-99.20	U1	S2	OK	1400	140.40	0.00	U1	All	OK
	7.500	1400	-140.40	-122.05	U1	S2	---	1400	140.40	0.00	U1	All	---
2	0.000	1400	-140.40	-56.49	U1	S2	---	1400	140.40	0.00	U1	All	---
	0.200	1400	-140.40	-42.04	U1	S2	OK	1400	140.40	0.00	U1	All	OK
	0.574	700	-77.35	-18.44	U1	S2	OK	1400	140.40	0.00	U1	All	OK
	1.079	700	-77.35	-0.03	U1	S1	OK	1400	140.40	11.55	U1	S3	OK
	1.453	0	0.00	0.00	U1	All	OK	1400	140.40	23.45	U1	Even	OK
	1.635	0	0.00	0.00	U1	All	OK	1400	140.40	27.82	U1	Even	OK
	2.250	0	0.00	0.00	U1	All	OK	1400	140.40	34.84	U1	Even	OK
	2.287	0	0.00	0.00	U1	All	OK	1400	140.40	34.90	U1	Even	OK
	2.865	0	0.00	0.00	U1	All	OK	1400	140.40	29.86	U1	Even	OK
	3.196	0	0.00	0.00	U1	All	OK	1400	140.40	22.33	U1	S2	OK
	3.496	700	-77.35	0.00	U1	All	OK	1400	140.40	13.08	U1	S2	OK
	3.941	700	-77.35	-11.52	U1	S3	OK	1400	140.40	0.00	U1	All	OK
	4.241	1400	-140.40	-29.00	U1	S3	OK	1400	140.40	0.00	U1	All	OK
	4.300	1400	-140.40	-32.74	U1	S3	OK	1400	140.40	0.00	U1	All	OK
	4.500	1400	-140.40	-46.35	U1	S3	---	1400	140.40	0.00	U1	All	---
3	0.000	1400	-140.40	-76.74	U1	S3	---	1400	140.40	0.00	U1	All	---
	0.200	1400	-140.40	-58.54	U1	S3	OK	1400	140.40	0.00	U1	All	OK
	0.731	700	-77.35	-16.39	U1	S3	OK	1400	140.40	0.00	U1	All	OK
	0.849	700	-77.35	-8.24	U1	S3	OK	1400	140.40	0.00	U1	All	OK
	1.380	0	0.00	0.00	U1	All	OK	1400	140.40	24.54	U1	Odd	OK
	2.160	0	0.00	0.00	U1	All	OK	1400	140.40	53.67	U1	Odd	OK

```
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2.963	0	0.00	0.00	U1 All	OK	1400	140.40	63.49	U1 Odd	OK
3.000	0	0.00	0.00	U1 All	OK	1400	140.40	63.43	U1 Odd	OK
3.840	0	0.00	0.00	U1 All	OK	1400	140.40	50.80	U1 Odd	OK
4.545	0	0.00	0.00	U1 All	OK	1400	140.40	23.32	U1 S3	OK
5.147	700	-77.35	-13.43	U1 Odd	OK	1400	140.40	0.00	U1 All	OK
5.198	700	-77.35	-16.99	U1 Odd	OK	1400	140.40	0.00	U1 All	OK
5.800	1400	-140.40	-65.79	U1 Odd	OK	1400	140.40	0.00	U1 All	OK
6.000	1400	-140.40	-84.55	U1 Odd	---	1400	140.40	0.00	U1 All	---

Longitudinal Beam Transverse Reinforcement Demand and Capacity
==

Section Properties

Units: dv (mm), Av/s (mm^2/mm), PhiVc, Vrmax (kN)

Span	dv	(Av/s)min	PhiVc	Vrmax
1	319.5	0.225	56.08	389.45
2	319.5	0.225	56.08	389.45
3	319.5	0.225	56.08	389.45

Beam Transverse Reinforcement Demand

Units: Start, End, Xu (mm), Vu (m), Av/s (kN/mm^2)

			Required				Demand	
Span	Start	End	Xu	Vu	Comb/Patt	Av/s	Av/s	
1	0.276	1.443	0.520	104.61	U1/Odd	0.313	0.313	
	1.443	2.366	1.443	75.31	U1/Odd	0.124	0.225	*8
	2.366	3.289	2.366	46.00	U1/Odd	0.000	0.000	
	3.289	4.211	3.289	16.70	U1/Odd	0.000	0.000	
	4.211	5.134	5.134	42.31	U1/S2	0.000	0.000	
	5.134	6.057	6.057	71.61	U1/S2	0.100	0.225	*8
	6.057	7.224	6.980	100.92	U1/S2	0.289	0.289	
2	0.276	1.014	0.520	58.90	U1/S2	0.018	0.225	*8
	1.014	1.508	1.014	43.21	U1/S2	0.000	0.000	
	1.508	2.003	1.508	27.51	U1/S2	0.000	0.000	
	2.003	2.497	2.003	11.81	U1/S2	0.000	0.000	
	2.497	2.992	2.992	23.34	U1/S3	0.000	0.000	
	2.992	3.486	3.486	39.04	U1/S3	0.000	0.000	
	3.486	4.224	3.980	54.74	U1/S3	0.000	0.225	*8
3	0.276	1.228	0.520	77.68	U1/S3	0.139	0.225	*8
	1.228	1.937	1.228	55.18	U1/S3	0.000	0.225	*8
	1.937	2.646	1.937	32.67	U1/S3	0.000	0.000	
	2.646	3.354	3.354	12.96	U1/Odd	0.000	0.000	
	3.354	4.063	4.063	35.46	U1/Odd	0.000	0.000	
	4.063	4.772	4.772	57.96	U1/Odd	0.012	0.225	*8
	4.772	5.724	5.480	80.46	U1/Odd	0.157	0.225	*8

NOTES:
*8 - Minimum transverse (stirrup) reinforcement governs.

Beam Transverse Reinforcement Details

Units: spacing & distance (mm).

Span	Size	Stirrups (2 legs each unless otherwise noted)
1	#10	10 @ 220 + <-- 2769 --> + 10 @ 220
2	#10	4 @ 211 + <-- 2472 --> + 4 @ 211
3	#10	8 @ 221 + <-- 2126 --> + 8 @ 221

Beam Transverse Reinforcement Capacity

Units: Start, End, Xu (m), Vu, PhiVn (kN), Av/s (mm^2/mm), Av (mm^2), Sp (mm)

			Required					Provided				
Span	Start	End	Xu	Vu	Comb/Patt	Av/s	Reqd/Min	Av	Sp	Av/s	PhiVn	
1	0.000	0.276	0.520	104.61	U1/Odd	-----	-----	-----	-----	-----	-----	
	0.276	2.366	0.520	104.61	U1/Odd	0.313	1.39	200.0	220	0.909	197.18	
	2.366	5.134	2.366	46.00	U1/Odd	0.000	0.00	-----	-----	-----	54.31	
	5.134	7.224	6.980	100.92	U1/S2	0.289	1.28	200.0	220	0.909	197.18	
	7.224	7.500	6.980	100.92	U1/S2	-----	-----	-----	-----	-----	-----	
2	0.000	0.276	0.520	58.90	U1/S2	-----	-----	-----	-----	-----	-----	
	0.276	1.014	0.520	58.90	U1/S2	0.018	0.08	200.0	211	0.949	203.30	*8
	1.014	3.486	1.014	43.21	U1/S2	0.000	0.00	-----	-----	-----	54.31	
	3.486	4.224	3.980	54.74	U1/S3	0.000	0.00	200.0	211	0.949	203.30	*8
	4.224	4.500	3.980	54.74	U1/S3	-----	-----	-----	-----	-----	-----	
3	0.000	0.276	0.520	77.68	U1/S3	-----	-----	-----	-----	-----	-----	
	0.276	1.937	0.520	77.68	U1/S3	0.139	0.62	200.0	221	0.903	196.22	*8
	1.937	4.063	4.063	35.46	U1/Odd	0.000	0.00	-----	-----	-----	54.31	
	4.063	5.724	5.480	80.46	U1/Odd	0.157	0.70	200.0	221	0.903	196.22	*8
	5.724	6.000	5.480	80.46	U1/Odd	-----	-----	-----	-----	-----	-----	

NOTES:

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   *8 - Minimum transverse (stirrup) reinforcement governs.

Slab Shear Capacity
===================
   Units: b, dv (mm), Xu (m), PhiVc, Vu(kN)
   Span        b        dv      Beta   Vratio         PhiVc            Vu            Xu
   ---- -------- -------- -------- -------- ------------ ------------ ------------
      1 --- Not checked ---
      2 --- Not checked ---
      3 --- Not checked ---

Material Takeoff
================
   Reinforcement in the Direction of Analysis
   ------------------------------------------
   Top Bars:        78.6 kg   <=>   4.37 kg/m     <=>  14.556 kg/m^2
   Bottom Bars:    197.8 kg   <=>  10.99 kg/m     <=>  36.633 kg/m^2
   Stirrups:        40.1 kg   <=>   2.23 kg/m     <=>   7.420 kg/m^2
   Total Steel:    316.5 kg   <=>  17.58 kg/m     <=>  58.609 kg/m^2
   Concrete:         2.2 m^3  <=>   0.12 m^3/m    <=>   0.400 m^3/m^2
```

4. Design Results Comparison and Conclusions

The following table shows the comparison between hand results and spBeam model results.

Table 5 – Comparison of the Continuous Beam Analysis and Design Results

Location		M_f, kN.m Before Redistribution		M_f, kN.m After Redistribution		$A_{s,req}$, mm^2	
		Hand	spBeam	Hand	spBeam	Hand	spBeam
Support A	Right Face	135.5	135.5	135.5	135.5	1,068	1,064
Midspan A-B		83.0	83.0	95.4	95.5	885	886
Support B	Left Face	147.4	147.4	122.1	122.0	923	925
	Right Face	70.6	70.6	56.5	56.5	395	364
Midspan B-C		23.5	23.5	34.9	34.9	300*	300*
Support C	Left Face	57.9	57.9	46.4	46.3	300*	300*
	Right Face	95.9	95.9	76.7	76.7	515	517
Midspan C-D		54.2	54.2	63.4	63.5	564	564
Support D	Left Face	84.5	84.5	84.5	84.5	589	587

* $A_{s,min}$ governs

The results of all the hand calculations used illustrated above are in precise agreement with the automated exact results obtained from the spBeam program.

The moment redistribution is often utilized for the investigation of existing structures for conditions such as change of use, additional loading, or verifying adequacy for the latest design code. In these conditions, any reserve capacity from existing reinforcement layout at mid-span (or support) of a span may be utilized to compensate for the inadequacy of the support (or mid-span) of the same span.

The moment redistribution can also be utilized in the design of a new structure. One such example of its application may help reduce the negative moment at an interior support and corresponding top reinforcement while increasing the positive moment at mid-span. The advantage of this may be the alleviation of the congestion of rebar at support top regions.

The calculation of moment redistribution is a tedious process especially while considering live load patterning as presented in this example. The procedure gets far more complicated if point loads or partial line loads are present. The spBeam software program performs the moment redistribution calculations with speed and accuracy.